Mathematical Modelling and High Bandwidth Allocation for Video Teleconference Service Traffic

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Abstract: The emerging high-speed networks, notably the Video Teleconference Service Traffic (VTST)-based Broadband ISDN, are expected to integrate through statistical multiplexing large numbers of traffic sources having a broad range of burstiness characteristics. A prime instrument for controlling congestion in the network is admission control, which limits calls and guarantees a grade of service determined by delay and loss probability in the multiplexer. We show, for general Semi Markovian traffic sources, that it is possible to assign a notional effective bandwidth to each source which is an explicitly identified, simply computed quantity with provably correct properties in the natural asymptotic regime of small loss probabilities. It is the maximal real eigenvalue of a matrix which is directly obtained from the source characteristics and the admission criterion, and for several sources it is simply additive. We consider both fluid and point process models and obtain parallel results. Numerical results show that the acceptance set for heterogeneous classes of sources is closely approximated and conservatively bounded by the set obtained from the effective bandwidth (EB) approximation. Also, the bandwidth-reducing properties of the Leaky Bucket regulator are exhibited numerically. For a source model of video teleconferencing due to *Tabatabai et al*. with a large number of states, the EB is easily computed. The equivalent bandwidths is bounded by the peak and mean source rates, and is monotonic and concave with respect to a parameter of the admission criterion. Coupling of state transitions of two related asynchronous sources always increases their EB.

Keywords: Effective Bandwidth, High Bandwidth, ON/OFF period, Single Source, Multiple Source, Semi Markov Process, Video Teleconference Service Traffic.

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1. INTRODUCTION

In statistical multiplexing, which is the core of VTST- based B-ISDN, We show that it is possible to assign a notional EB to each source which reflects its characteristics, including burstiness and the service requirements. The sources are given great generality; there are no restrictions on dimensions, homogeneity or time reversibility. Yet, it is shown that the EB is an explicitly identified quantity with provably correct asymptotic properties which can be obtained from simple and standard computations. In numerical evaluations of realistic admission control, approximations based on EB perform very well. Importantly, the EB of a source is independent of traffic submitted by other sources to the multiplexer. This fact makes the complexity of computing the EB depend only on the source, not system, dimension; it also offers the promise of decentralized estimation from measurements and enforcement of the EB. Specifically, we show that the EB of a Semi Markovian source is the maximal real eigenvalue of a matrix, derived from the source parameters, network resources and service requirements, with dimension equal to the number of source states. Two parallel sets of results are obtained: one for a fluid model of statistical multiplexing with Markov-modulated fluid sources and the other for queues and point processes in which the traffic sources are, for example, Semi Markov-modulated Poisson (SMMP). The results extent the recent and important results of Hunt and Gibbens [22], who consider heterogeneous ON/OFF fluid sources which alternate between exponentially distributed periods of transmission at the peak rate and quiescence. Even for the case of ON/OFF fluid sources, the results here shed new light on origins of key expressions in [22] and also on the expressions used there from earlier work by Mitra *et al.*[32] and Kosten[26].

The imminence of new services with a broad range of burstiness characteristics and their integration through statistical multiplexing has focused attention on call admission as the prime instrument of rate-based congestion control. For a survey of issues, approaches, and analyses, see [35] and, for a more recent update,[36]. By preventing admission to an excessive number of calls or sources to the multiplexer, call admission policies strive to

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strike a balance between grade of service(as determined by delay and cell loss probability, for instance)and efficient use of network resources. Designs based on peak rates and mean rates are two extremes; hence, it is no accident that the EB of a source is proven here to be bounded by these two rates. Designers have gravitated toward the concept of EB because it promises simplicity and the hope that it might be a bridge to familiar circuit-switched network designs. It should be emphasized that the notion of EB is intimately connected with admission control and the associated service requirements. Consequently, it is determined by the source characteristics in conjunction with the admission criteria. Hunt and Gibbens [22], Kelly[23], Ahmadi *etal.*[2], Warland and Kesidis [43], Chang[7],and Whitt[44] offer different approaches to EB. Kelly finds EB for *G1*/*G*/1 queues. Guerin *et al.* independently obtain the formulas in [22] through insightful interpretations of the results in [32] and extend them through heuristics. Both [43] and [7] consider the general problem of the existence of an EB of stationary and ergodic sources. Kesidis and Walrand take a large deviation approach to determine the EB, where the admission criteria is identical to that in this paper. As is typical with this approach, they give the EB in terms of the solution of a substantial variational problem; this problem is solved only for two-state ON/OFF sources. There is an intimate connection between the behavior of tail probabilities of queue lengths and EB. Whitt gives a detailed treatment of this connection for multiclass queues; see also Sohraby[38]. Chang develops bounding techniques for tail behavior of queues in networks and among other results, explicitly connects the bounds for two-state ON/OFF sources to the EB in[22].Other approaches to admission control and EB based on loss networks are due to Hui [21] and Lindberger[28].

In the model of statistical multiplexing considered, each fluid source is characterized by ($\boldsymbol{M}$, λ). Where $\boldsymbol{M}$ is the infinitesimal generator of a controlling Semi Markov Chain. The source generates fluid at the constant rate λ_s when in state s. The mean source rate is denoted by $\overline{\lambda}$ and the peak rate by $\hat{\lambda}$. The multiplexing buffer is serviced by a channel of constant capacity, or rate, c. Let $G(B)$ denote the stationary P[$X \geq B$] where X represents the random buffer content and interpret $G(B)$ to be the overflow probability for a buffer of size B. For given B and p , let the service requirement be $\{G(B) \leq p\}$,which is also taken to be the admission criterion. We think of p as being small, of the order of 10^{-9}.

Now consider the statistical multiplexing system in which there is only a single-source ($\boldsymbol{M}$, λ).There are no restrictions on the dimensions of this source. We show that in the asymptotic regime where $p \to 0$ and $B \to \infty$ in such a manner that $\frac{\log p}{B} \to \zeta \in [-\infty, 0]$, the admission criterion is satisfied if $e < c$ and violated if $e > c$. We call e the EB and show that it is the maximal real eigenvalue of the matrix $[\boldsymbol{R} - \frac{1}{\zeta}\boldsymbol{M}]$ where $\mathbf{R}$ = diag (λ). The EB e depends on ($\boldsymbol{M}$, λ), of course, and on the buffer and overflow probability only through ζ .

Now suppose that the single source just considered is, in fact, the aggregate of K arbitrary sources, ($\boldsymbol{M}^{(k)}$, $\lambda^{(k)}$) ($1 \leq k \leq K$). We obtain a result of remarkable simplicity: the EB $e = \sum e^{(k)}$, where $e^{(k)}$ is the EB of the source ($\boldsymbol{M}^{(k)}$, $\lambda^{(k)}$)computed as if it is a single source in the system.

In all important respects, the results carry over to the framework of queues and point processes. The source characterization differs only in that λ_s is the rate of the Poisson stream which is generated by the source when in state s. The EB of the single-source ($\boldsymbol{M}$, λ) in the multiplexing system is now the maximum real eigenvalue of $\left(\frac{1}{e^{\zeta}}R - \frac{1}{1-e^{\zeta}}M\right)$ for ζ defined as before. In the rest of the paper, we focus on the fluid model and handle the queueing model exclusively in Section 7.

We show that, in the fluid model, the EB decreases monotonically with increasing ζ from $\hat{\lambda}$ at $\zeta = -\infty$ to $\bar{\lambda}$ at $\zeta = 0$. We also show that the coupling of state transitions from two asynchronous sources with identical infinitesimal generators and proportional rate vectors always leads to an increase in EB. Examples show that is not true, in general, if the rate vectors are arbitrary. These facts are important if the pricing of network services is based on EB.

The following is an observation on the EB which may be useful for its estimation from measurements. Consider a test bed in which the source supplies a buffer which is serviced by a channel of variable capacity c. The EB e is that value of c for which the asymptotic slope of log $G(x)$ equals ζ .

The additive from in the EB of K sources has simplifying consequences for the call admission problem with multiple heterogeneous classes of sources. We want $A(B, p) = \{\boldsymbol{K} = (K^{(1)} \ldots\ldots\ldots\ldots K^{(j)} : G_K(B) \leq p)$. The asymptotic result is that $A(B, p)$ is essentially the simplex $\sum e^{(j)} K^{(j)} < c$, where $e^{(j)}$ is the EB of a single source of class j.

This asymptotic result motivates the approximation $\sum e^{(j)} K^{(j)} < c$ to the acceptance set in real, nonasymptotic cases. We have tested the goodness of this approximation for a variety of classes which display different burstiness aspects. Note that both the exactly calculated and the approximate acceptance sets are not exactly simplexes because of the integrality of $\{K^{(j)}\}$. It is our experience (reported in section 5) that the approximation is uniformly good provided that B is at least moderately large and, often, even when B is small. Also, we have observed that, importantly, the EB approximation provides a conservative bound on the acceptance set.

Since so much emphasis has been given in prior work to two-state ON/OFF sources, it is worth noting some of the reasons for considering higher-dimensional sources. First, as has been noted by Tabatabai *et al*. [41] for video teleconference traffic, two-state sources do not capture essential traffic features. In fact, Tabatabai *et al*. used models for individual sources which have over 600 states. See also Anastassiou *et al*. [3] for other high-order video source models .Second, higher-order models are concomitant with the need to design call admission in conjunction with traffic monitoring and regulation. Mitra and Elwalid [12] have studied regulated traffic and its approximate Semi Markovian characterization, which, in the case of the simplest class of regulators, has dimension one more than that of the original source model. Finally, even for ON/OFF sources, there is considerable interest in the effects of variability of the ON and OFF periods [6].Such analysis requires higher-dimensional source models.

Results reported in Sections 5 and 6 specifically address the points raised previously. In section 6, we show that the EB of a video teleconference source model with a large number of states one recommended by Heyman *et at*., can be quite easily calculated. Also, in Section 5 we numerically examine the bandwidth –reducing features of a Leaky Bucket regulator. These results are in agreement with Konstantopoulos and Anantharaman [25], from where it can be inferred that the EB of the output of the Leaky Bucket is monotonic with respect to the regulator's parameter. Finally, Section 5 gives a simple source model with four states which accommodate hyper exponentially distributed ON and OFF periods and also presents data on their influence on EB.

The mathematical developments belong to two different categories: the first has to do with the analysis of just the single source, and the second with the algebraic decompositions which give the additive from to the EB of several sources. The essential steps in the first category are broadening of the scope of the standard eigenvalue problem by introducing an inverse eigenvalue problem and investigating the growth properties of the maximum real eigenvalue with respect to a parameter in the problem. It is to the inverse problem that we bring to bear a fundamental result due to Cohen [10], [11] and Friedland [16] on the convex behavior of the maximum real eigenvalue of essentially nonnegative matrices with respect to all diagonal elements. In the second category, the algebraic theory which gives the important decompositions is based on Kronecker representations and separability, which has its antecedents in the work of Mitra *etal*.[32], Kosten [26], [27], Mitra [29], Elwalid and Stern [14] and Stern *etal*.[40].

2. PRELIMINARIES

This section, which is in three parts, begins by giving some basic background facts about the statistical multiplexing system. Computation of the spectral expansion of the system's stationary distribution involves a standard eigenvalue problem. The second part of this section (Section 2.2) points out that, in this paper, it will be necessary to broaden the scope of the eigenvalue problem by introducing a parameter (the channel capacity) and view the eigenvalues as functions of this parameter and, also to look at the inverse problem, which turns out to be an eigenvalue problem as well. Finally, the last part of this section (Section 2.3) presents some known facts about essentially nonnegative matrices and the maximal real eigenvalues critical for the analytic development in subsequent sections.

2.1 The Statistical Multiplexing System

The statistical multiplexing system consists of a buffer which is supplied by various statistically independent Semi Markov-modulated fluid sources and serviced by a channel of constant capacity, i.e., rate c. For our purposes, it will suffice to lump the source description into a single aggregate Semi Markov-modulated fluid source with state space S and irreducible generator $\boldsymbol{M}$. (In Section 4, it will be necessary to consider the detailed structure of $\boldsymbol{M}$ implied by the presence of several lower-order sources) this aggregate source generates fluid at the constant rate λ_s when in state s ($s \in S$). Let $\boldsymbol{\lambda}=\{\lambda_s | s \in S\}$. Thus, the aggregate source is characterized by $(\boldsymbol{M}, \boldsymbol{\lambda})$. We also let the rate matrix $\mathbf{R}$ = diag $(\boldsymbol{\lambda})$.

Let $\sum$ and X denote the stationary aggregate-source state and buffer content, respectively. Let the stationary state distribution of the multiplexing system be denoted by $\boldsymbol{\pi}$ (x), where $\boldsymbol{\pi}$ (x) $= \{\pi_s(x) \mid s \in S\}$ and

$$\pi_s(x) \triangleq \mathrm{P}\,(\textstyle\sum = s,\ X \le x) \quad (s \in S,\ 0 \le x \le \infty) \qquad (1)$$

The governing system of differential equations is

$$\frac{d}{dx}\boldsymbol{\pi}(x)\,\boldsymbol{D} = \boldsymbol{\pi}(x)\,\boldsymbol{M} \qquad (0 \le x \le \infty) \qquad (2)$$

Where $\boldsymbol{D} \triangleq \mathbf{R} - c\boldsymbol{I}$ and $\boldsymbol{I}$ are the identity matrixes and the diagonal element $D_{ss} = (\lambda_s - c)$ is the drift or rate of change in the buffer content when the source is in state *s*. Hence, we call $\boldsymbol{D}$ the drift matrix.

The stationary probability vector for the aggregate source is denoted by $\boldsymbol{\omega}$; hence, $\boldsymbol{\omega M} = 0$ and $\langle \boldsymbol{\omega}, 1 \rangle = 1$. The symbol $\langle .\,,\, . \rangle$denotes the inner product of vectors and 1 is the vector in which all elements are unity. The ergodicity condition is

$$\bar{\lambda} < c \qquad (3)$$

Where the mean source rate is

$$\bar{\lambda} \triangleq \langle \boldsymbol{\lambda}, \boldsymbol{\omega} \rangle \qquad (4)$$

We denote the peak source rate by$\hat{\lambda}$, i.e., $\hat{\lambda} \triangleq \max_s \lambda_s$. To rule out the trivial case in which there is never any accumulation in the multiplexing buffer, we assume that $c < \hat{\lambda}$.

Since the stationary state distribution is a bounded solution, it has the spectral representation

$$\pi(x) = \sum_{i:\mathrm{Re}\, z_i < 0} a_i \phi_i e^{z_i x} + \omega \qquad (5)$$

Where (z_i, $\boldsymbol{\phi}_i$) is an eigenvalue/eigenvector pair. Such pairs are solutions to the eigenvalue problem

$$Z\,\boldsymbol{\phi}\,\boldsymbol{D} = \boldsymbol{\phi}\,\boldsymbol{M} \qquad (6)$$

The eigenvalues with negative real parts are indexed as

$$0 > Re\, z_1 \ge Re\, z_2 \ge Re\, z_3 \ge \ldots \qquad (7)$$

If z_1 is real and $z_1 > \mathrm{Re}\ z_i$ for all $i > 1$, Then z_1 is called the *dominant eigenvalue.*

In the spectral expansion, the coefficients $\{a_i\}$ are obtained by solving a system of linear equations which are obtained by the following boundary conditions (see, for instance,[29])

$$D_{ss} > 0 \Longrightarrow \pi\ (s, 0) = 0 \qquad (8)$$

It is known that the number of such conditions exactly equals the number of eigenvalues with negative real parts.

Let the stationary buffer overflow distribution be given by $G(x)$, i.e.,

$$\begin{aligned} G(x) &= \mathrm{P}\,(X \ge x) \\ &= 1 - \langle \boldsymbol{\pi}(x), \mathbf{1} \rangle \\ &= \sum_{i \ge 1} a_i \langle \phi_i, 1 \rangle e^{z_i x} \end{aligned} \qquad (9)$$

If z_1 is the dominant eigenvalue, then

$$G(x) \sim a_1 \langle \phi_i, 1 \rangle\ e^{z_1 x} \text{ as } x \to \infty \qquad (10)$$

Note that

$$z_1 = \lim_{x \to \infty} \frac{\log G(x)}{x} \qquad (11)$$

Plots of log $G(x)$ versus x approach linearity as x increases, and the slope approaches Z_1.

2.2 Inverse Eigenvalue Problem

Consider the eigenvalue problem in (6)

$$z\, \emptyset\, (\mathbf{R} - c\boldsymbol{I}) = \emptyset\, \boldsymbol{M} \qquad (12)$$

It is convenient to extend the scope of the problem by considering c to be a variable parameter and the eigenvalues to be functions of c, $z(c)$. The inverse problem requires c to be obtained for given z. The key fact in this connection is that this inverse problem is also an eigenvalue problem. For, writing $c = g(z)$, we obtain from (12)

$$g(z)\, \emptyset = \emptyset\, \boldsymbol{A}\,(z) \qquad (13)$$

Where

$$\boldsymbol{A}(z) = \boldsymbol{R} - \frac{1}{z}\boldsymbol{M} \qquad (14)$$

That is, $g(z)$ is an eigenvalue of the matrix $\boldsymbol{A}(z)$ in which z is a parameter.

The inverse eigenvalue problem, its maximal real eigenvalue and the behavior of this eigenvalue as a function of z will be important in the subsequent development.

2.3 Essentially Nonnegative Matrices

A real matrix with nonnegative elements off the main diagonal is called essentially nonnegative. The matrix $\boldsymbol{A}\,(z)$ in (14) is essentially nonnegative for real and negative z. Since $\boldsymbol{M}$ is irreducible, so is $A(z)$. By adding $\sigma\boldsymbol{I}$ to $A(z)$, where

$$\sigma > \left\{ \max_i \left(\frac{1}{z} M_{ii} - \lambda_i \right) \right\} \qquad (15)$$

We obtain a matrix which is nonnegative and eigenvalues which are the eigenvalues of $A(z)$ shifted by σ. Thus, the Perron-Frobenius Theory [17],

[20], applies to $[\boldsymbol{A}(z) + \sigma\boldsymbol{I}]$, and we can infer the following for matrix $\boldsymbol{A}(z)$ $(z<0)$.

Result 1: There exists a real Eigen value $g_1(z)$ of the matrix $\boldsymbol{A}(z)$ such that to $g_1(z)$ can be associated a real vector $\boldsymbol{\phi}_1$, where $\boldsymbol{\phi}_1 > 0$(element wise) and $\min_s \lambda_s < g_1(z) < \hat{\lambda}$. If $g(z)$ is any other eigen value then Re $g(z) < g_1(z)$. The eigenvalue $g_1(z)$ is simple.

The eigenvalue $g_1(z)$ is referred to as the *maximal real eigenvalue*. The maximal real eigenvalue of an essentially nonnegative matrix need not be the eigenvalue with the largest modulus. The upper and lower bounds on $g_1(z)$ in Result 1 correspond to the maximum and minimum row sums of $\boldsymbol{A}(z)$.

A result due to Cohen [9], Theorem 1, Corollary 2] allows the lower bound in Result 1 to be sharpened. Although Cohen's result is stated for nonnegative matrices, it is readily adapted to $\boldsymbol{A}(z)$ $(z<0)$.(Recall from (4) that $\bar{\lambda}$ is the mean source rate.)

Result 2: $\bar{\lambda} \leq g_1(z)$.

We shall also need the following fundamental and important result due to Cohen [10],[11]and Friedland [16]:

Result 3: The maximal real eigenvalue of $(\boldsymbol{A}+\boldsymbol{\Delta})$ is a strictly convex function of $\boldsymbol{\Delta}$, where $\boldsymbol{A}$ is any irreducible, essentially nonnegative matrix and $\boldsymbol{\Delta}$ is any diagonal matrix which diagonal elements which are not all identical. That is,

$$r((1-h)\boldsymbol{A}+h(\boldsymbol{A}+\boldsymbol{\Delta})) < (1-h)r(\boldsymbol{A})+hr(\boldsymbol{A}+\boldsymbol{\Delta}) \quad (0<h<1) \qquad (16)$$

Where r (.) is the maximal real eigenvalue. This result is equivalent to the positive definiteness of the Hessian $\boldsymbol{H} = (H_{ij})$,where

$$H_{ij} = \frac{\partial^2 r(A+\Delta)}{\partial\Delta_{ii}\partial\Delta_{jj}}.$$

The antecedents of this result are rich and varied. Cohen [11] showed weak convexity by a Feynman-Kac formula for the maximal real eigenvalue of nonnegative matrices. Friedland [16], who was the first to show strict convexity, used a variational characterization by Donsker and Varadhan for the maximum real eigenvalue. Cohen [10] also showed strict convexity by using the Trotter product formula and a theorem on log convexity due to Kingman [24] as extended by Seneta [37].

Finally, we shall need recourse to a well-known result on nonnegative matrices [17],[20].

Result 4: The maximal real eigenvalue of a nonnegative, irreducible matrix increases when any matrix element increases.

This result remains intact when the nonnegativity of the matrix is substituted by essential nonnegativity.

Note that increasing z in $\boldsymbol{A}(z) = [\mathbf{R} - \frac{1}{z}\boldsymbol{M}]$ has the effect of increasing the nonzero off-diagonal elements , and of decreasing the diagonal elements of the matrix. Hence, this result by no means implies the monotonicity of the maximal real eigenvalue $g_1(z)$ with respect to z. This topic, which is of central importance in our study, is examined in the next section.

3. SINGLE SOURCE

This section on the single source plays a pivotal role in this discussion. First, when the single source in the multiplexing system is allowed to be of arbitrary dimension (as it is in this section), then it can be construed to be the aggregate of many lower-order sources and the many-source problem becomes an extension of the single-source problem in which the new element is the algebraic exploitation of the structure implied by the presence of many sources. Such an extension is undertaken in the next section. Second, the qualitative properties of the eigenvalues, such as monotonicity and convexity, are established in Section 3.1. Third, the asymptotic view of the admission control problem is introduced in section 3.2. The result identifying EB of the source as the maximal real eigenvalue of a simple matrix is proven there. Finally, in Section 3.3 we show that the EB is a monotonic increasing and convex function of all state- dependent rates of the source. A corollary to this result is that, whenever we couple the state transitions of two sources having identical generators for their controlling Markov chains and proportional rate vectors, the effect is to increase the EB.

Let the source be characterized by $(\boldsymbol{M}, \boldsymbol{\lambda})$ where $\boldsymbol{M}$ is any irreducible infinitesimal generator. The number of states in the controlling Markov chain, which is also the dimension of $\boldsymbol{M}$ and $\boldsymbol{\lambda}$ is N .The system considered in this section consists of this source supplying a buffer which is serviced by a channel of capacity (rate) c. The admission control problem is to characterize sources for which the admission criterion $\{G(B) \leq p\}$ is satisfied.

3.1 Monotonicity of the Maximal Real Inverse Eigenvalue Problem

We examine the maximal real inverse eigenvalue problem (13). Recall that, in this problem, the parameter is z and the eigenvalue is $g(z)$:

$$g(z)\,\boldsymbol{\phi} = \boldsymbol{\phi}\boldsymbol{A}(z) \qquad (17)$$

Where

$$\boldsymbol{A}(z) = \boldsymbol{R} - \tfrac{1}{z}\boldsymbol{M} \qquad (18)$$

Making use of Result 1 of section 2.3, the solutions to (17) for $z < 0$ are indexed thus:

$$g_1(z) > \mathrm{Re}\,g_2(z) \geq \mathrm{Re}\,g_3(z) \geq \ldots \qquad (19)$$

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The maximal real eigenvalue is $g_1(z)$. From Results 1 and 2

$$\bar{\lambda} \le g_1(z) \le \hat{\lambda} \quad (z < 0) \tag{20}$$

We will find it convenient more than once to complement (17) by the form which is obtained by multiplying (17) by $(-z)$:

$$(-zg(z))\, \boldsymbol{\phi} = \boldsymbol{\phi}[(-z)\, \boldsymbol{R} + \boldsymbol{M}] \tag{21}$$

The matrix $(-z)\ \boldsymbol{A}\ (z)$ on the right side remains essentially nonnegative and hence has a maximal real eigenvalue, which we denote by $r(z)$. Note that $r(z) = -zg_1(z)$.

The form in (21) is useful for obtaining the limiting value of $g_1(z)$ as $z \to 0$. From Result 1 of Section 2.3, we know that $g_1(z)$ and $r(z)$ are simple; consequently, standard perturbation analysis applies [45]. Expanding $r(z)$ and ϕ in power series, $r(z) = r_0 + r_1 z + r_2 z^2 + \ldots$, and $\phi(z) = \phi_0 + \phi_1 z + \phi_2 z^2 + \ldots$, we obtain from (21) $r_0 = 0$, $\boldsymbol{\phi}_0 = \boldsymbol{\omega}$, and $r_1 = -\langle \boldsymbol{\lambda}, \boldsymbol{\omega} \rangle = -\bar{\lambda}$. Hence

$$g_1(0) = -\left(\lim_{z \to 0} \frac{r(z)}{z}\right) = -r_1 = \bar{\lambda} \tag{22}$$

When $z \to -\infty$, it is apparent from (17) and (18) that $g_1(z) \to \hat{\lambda}$. To recapitulate, we have that the maximal real eigenvalue $g_1(z)$ of $A(z)$ satisfies, for $z < 0$,

Proposition 1

$$g_1(0) = \bar{\lambda} \text{ and } g_1(-\infty) = \hat{\lambda} \tag{23}$$

Hence, the bounds in (20) are tight.

The effect of decreasing z is to increase all the diagonal elements of $[(-z)\ \boldsymbol{R} + \boldsymbol{M}]$ for which the corresponding diagonal elements of $\boldsymbol{R}$ are nonzero, while all other diagonal elements and all off-diagonal elements are not affected. Hence, it follows from Result 4 of Section 2.3 that

$$\frac{\partial r}{\partial z} < 0 \qquad (z < 0) \tag{24}$$

Next, we use Result 3 to establish the convexity of $r(z)$. Let $z_2 < z_1 < 0$ and $0 < h < 1$. In Result 3, identify $\boldsymbol{A}$ with $[(-z_1)\ \boldsymbol{R} + \boldsymbol{M}]$ and $\boldsymbol{\Delta}$ with $(z_1 - z_2)\ \boldsymbol{R}$ to obtain

$$r\{(1-h)\, z_1 + hz_2\} < (1-h)\, r(z_1) + hr(z_2) \tag{25}$$

This condition is equivalent to

$$\frac{\partial^2 r}{\partial z^2} > 0 \quad (z < 0) \tag{26}$$

Proposition 2 The maximal real eigenvalue $r(z)$ of the essentially nonnegative matrix $(-z)\ A(z) = [(-z)\ \boldsymbol{\Lambda} + \boldsymbol{M}]$ $(z < 0)$ is a monotonically decreasing convex function. Moreover, $r(z) \sim \hat{\lambda}\, |z|$ as $z \to -\infty$, and $r(0) = 0$.

Recalling that $r(z) = -zg_1(z)$, it follows from (24) and (26) that, for $z < 0$,

$$g_1(z) + zg'_1(z) > 0 \tag{27}$$

$$2g'_1(z) + zg''_1(z) < 0 \tag{28}$$

Proposition 3 The maximal real eigenvalue $g_1(z)$ of the essentially nonnegative matrix $A(z) = \left[\mathbf{R} - \frac{1}{z}\, \boldsymbol{M}\right]$ is monotonic, decreasing with increasing z,

$$g'_1(z) < 0 \qquad (z < 0) \tag{29}$$

Proof: From (28), $g'_1(z) < 0$ when $|z|$ is small and from (27), $g'_1(z) < 0$ when $|z|$ is large. Now suppose that there exists some z for which (29) is false. Then, there exists intervals in which the sign of $g'_1(z)$ is uniform and in neighboring intervals the signs are opposite. In particular, there must exist a common endpoint to two such contiguous intervals, say z_1 $(-\infty < z_1 < 0)$, where a local maximum is reached, i.e.,

$$g'_1(z_1) = 0, \text{ and } g''_1(z_1) \le 0 \tag{30}$$

Notice that $2g'_1(z_1) + z_1 g_1''(z_1) \ge 0$, which contradicts (28).

We can also show that $g_1(z)$ is a concave function ; the proof is omitted.

Standard perturbation analysis readily yields an expression for $g'_1(z)$. In (22), let z be perturbed to $z + \epsilon$ and consider an expansion in powers of ϵ, $g_1(z + \epsilon) = g_1(z) + \Sigma\, \epsilon^i g_{1i}(z)$ and similar expansions for the left and right eigenvectors. By equating coefficients of ϵ^0 and ϵ^1, we obtain

$$g'_1(z) = \frac{1}{z^2} \frac{\boldsymbol{\phi}(z) \boldsymbol{M} \boldsymbol{\psi}(z)}{\boldsymbol{\phi}(z) \boldsymbol{\psi}(z)} \tag{31}$$

where $\phi(z)$ and $\psi(z)$ are, respectively, the left (row) and right (column) real eigenvectors of $\boldsymbol{A}(z)$ corresponding to the eigenvalues $g_1(z)$.

Notice that when the Semi Markov chain with generator $\boldsymbol{M}$ is time reversible, then $\boldsymbol{M}$ is essentially symmetric and negative semi-definite. Also, the form in (31) immediately shows that $g'_1(z) < 0$ for $z < 0$ ($g'_1(0)$ is more delicate). This result was established by Elwalid and Stern [14].

An ON/OFF source with exponentially distributed ON and OFF periods is obtained by setting $\lambda_1 = 0$ in the following representation:

$$\boldsymbol{M} = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix} \text{ and } \quad \boldsymbol{\lambda} = [\,\lambda_1 \ \ \lambda_2\,] \tag{32}$$

The reader may verify that

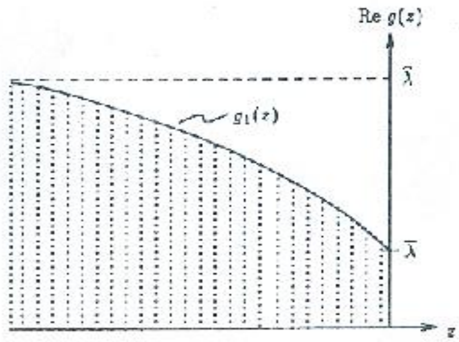
$$g_1(z) = \frac{1}{2z}[(\lambda_1 + \lambda_2)z + \beta + \alpha] - \frac{1}{2z}\sqrt{\{(\lambda_1 + \lambda_2)z + \alpha + \beta\}^2 - 4(\lambda_1\lambda_2 z^2 + \beta\lambda_1 z + \alpha\lambda_2 z)} \tag{33}$$

This expression is central to Mitra *etal.* [32] and Kosten [26]. Direct differentiation of (36), the procedure followed by Hunt and Gibbens [22], confirms that $g'_1(z) < 0$ for $z < 0$.

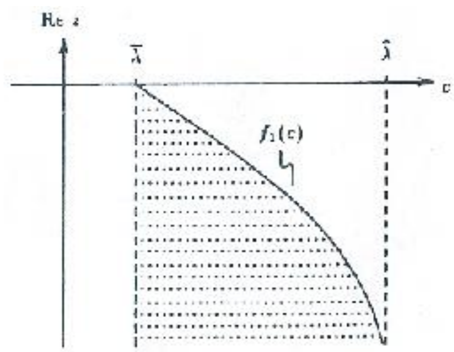
Fig. 1(a) is a sketch incorporating the results in Propositions 1 and 3.

The key duality between the two extremal eigenvalues of interest in the direct and inverse eigenvalue problems is now given.

Fig 1. (a) The shaded region contains the real parts of the inverse eigen value problem.



(b) The shaded region contains the real parts of all the solutions with negative real parts to the eigen value problem



Proposition 4 For $c \in (\bar{\lambda}, \hat{\lambda})$ the dominant eigenvalue z_1 is the unique solution in $(-\infty, 0)$ satisfying

$$g_1(z_1) = c \tag{34}$$

i.e., the dominant eigenvalue is the unique parameter in $\left[\mathbf{R} - \frac{1}{z}\boldsymbol{M}\right]$ for which the maximal real eigenvalue is c.

Proof: Since $g_1(z)$ is monotonic, strictly decreasing for $z \in [-\infty, 0]$ and takes values between $(\bar{\lambda}, \hat{\lambda})$, [36] has a unique solution. If z_2 is any other real solution to $g(z) = c$, then $z_2 < z_1$. If $z_2 > z_1$, then

$$c = g(z_2) \le g_1(z_2) < g_1(z_1) = c$$

a contradiction. It only remains to show that the dominant eigenvalue is real. The proof is similar to that maximal real eigenvalue in the inverse eigenvalue problem is real and we omit the detailed proof.

Denote the unique inverse of g_1 in $[-\infty, 0]$ by f_1, i.e. $f_1(g_1(z)) = z$ $(z < 0)$.

Hence, f_1 maps$[\bar{\lambda}, \hat{\lambda}]$ to $[0, -\infty]$ and

$$z_1 = f_1(c) \qquad (\bar{\lambda} \le c \le \hat{\lambda}) \tag{35}$$

It is easily seen from an application of the chain rule that

Proposition 5

$$\frac{df_1}{dc} < 0 \qquad (\bar{\lambda} < c < \hat{\lambda}) \tag{36}$$

i.e., the dominant eigenvalue z_1 is a monotonic, strictly decreasing function of the channel capacity c for $c \in (\bar{\lambda}, \hat{\lambda})$.

These results are incorporated in fig. 1(b).

3.2 Small Overflow Probabilities, Large Buffers

We now consider the admission control problem for an asymptotic regime in which the buffer overflow probabilities (p) is small, say of the order of 10^{-9}. For the scaling in this asymptotic regime to be meaningful, it is of course necessary to have large buffers. Enough is already known [see(10)] about the qualitative manner in which overflow probabilities scale with buffer size (B) to arrive at the following natural asymptotic regime, which is also the regime considered by Hunt and Gibbens [22]. Let $B \to \infty$, and also $p \to 0$, in such a manner that

$$\log p = \zeta B + O(1) \tag{37}$$

Where $\zeta \in [-\infty, 0]$ is any $O(1)$ parameter. Hence, $\dfrac{\log p}{B} \to \zeta$.

Since in this section we are considering a system with a single source, the multiplexing is nonexistent. The problem here is to characterize sources which supply a system with a buffer of size B and channel capacity c, and for which the buffer overflow probability $G(B)$ does not exceed p in the aforementioned asymptotic regime.

Proposition 6 Let the admission criterion be $G(B) \le p$ Suppose $B \to \infty$ and $p \to 0$ in such a manner that $\dfrac{\log p}{B} \to \zeta \in [-\infty, 0]$. If $g_1(\zeta) < c$, then the admission criterion is satisfied. If $g_1(\zeta) > c$, then the admission criterion is violated, where $g_1(\varsigma)$ is the maximal real eigenvalue of $A(\zeta) = \mathbf{R} - \dfrac{1}{\zeta}\boldsymbol{M}$.

Proof:

From(9), $$G(B) = \sum_{i \ge 1} a_i \langle \phi_i, 1 \rangle e^{z_i B} \tag{38}$$

Recall that z_1 is the dominant eigenvalue, so that $z_1 > \mathrm{Re}\, z_i$ for all $i>1$ and [see (34)] $c = (g_1(z_1)$. So

$$\frac{G(B)}{p} = a_1\langle \boldsymbol{\phi}_1, \mathbf{1}\rangle\, e^{(z_1-\zeta)B}\,[1+o(1)] \text{ as } pB \to \infty \tag{39}$$

Now, from Proposition 3, $g_1(z)$ decreases as z increases ; hence, if $g_1(\zeta) < c$, then $z_1 < \zeta$ [see Fig. 1(a)] and $\left(\frac{G(B)}{p}\right) \to 0$ as $B \to \infty$. Therefore, the admission criterion is satisfied. Similarly, if $g_1(\zeta) > c$ then $z_1 > \zeta$ and $\left(\frac{G(B)}{p}\right) \to \infty$, so the admission criterion is violated.

This result justifies the use of the term "effective bandwidth "for the quantity $g_1(\zeta)$. We let $e = e(\boldsymbol{M}, \boldsymbol{\lambda}; B,p)$ denote the EB of the source $(\boldsymbol{M},\lambda)$ in the system for which the admission criterion is $G(B) \leq p$. That is,

$$e(\boldsymbol{M}, \boldsymbol{\lambda}; B,p) = g_1(\zeta) \tag{40}$$

Where $g_1(\zeta)$ is the maximal real eigenvalue of the matrix $(\boldsymbol{\Lambda} - \frac{1}{\zeta}\boldsymbol{M})$ and $\zeta = \frac{\log p}{B}$.

The fact that B and p determine e only through ζ is a consequential fact which simplifies and benefits the design process (see the discussion in [22]). Note that the EB is independent of the channel capacity. From (23), (29) and (40), we observe that the EB decreases monotonically with increasing ζ from the peak source rate $\hat{\lambda}$ when $\zeta = -\infty$ to the mean source rate $\bar{\lambda}$ when $\zeta = 0$.

There are several effective numerical algorithms for calculating the maximal real eigenvalue and the Perron root of nonnegative matrices, such as the inverse iteration method (see [45] and [15]). Recall from (15), $(\mathbf{R} - \frac{1}{\zeta}\boldsymbol{M} + \sigma\boldsymbol{I})$ is a nonnegative matrix for

$$\sigma > \left\{\max_i\left(\frac{1}{\zeta}M_{ii} - \lambda_i\right)\right\}^+$$

The discussion in Section 2.2 on the inverse eigenvalue problem indicates an interpretation of $g_1(\zeta)$ which may be quite useful for obtaining the EB from measurements. Consider a testbed in which the source supplies a buffer which is emptied by a channel of (variable) capacity c. The effective bandwidth e is that value of c for which the asymptotic slope of $\log G(x)$ equals ζ .

3.3 Monotonically and Convexity of the EB with Respect to Source Rates

Here we investigate the influence of the source rates $\lambda = (\lambda_1, \lambda_2, \ldots\ldots \lambda_N)$ on the EB of the source, $e(\boldsymbol{M}, \boldsymbol{\lambda}; B,p)$.First, we establish that with $\boldsymbol{M}$,B, and p held fixed, the EB is strictly monotonic (increasing with each increasing λ_i)and also convex in $(\lambda_1, \lambda_2, \ldots\ldots \lambda_N)$. Next, these properties are used to obtain the following in-equality.

$$e(\boldsymbol{M}, \boldsymbol{\lambda}; B,p) > e(\boldsymbol{M}, a\boldsymbol{\lambda}; B,p) + e(\boldsymbol{M}, (1-a)\lambda; B,p) \tag{41}$$

for all $a \epsilon (0,1)$. The right quantity is the sum of the EBs of two sources which have identical controlling generator $\boldsymbol{M}$ for their Markov chains, proportional rate vectors and are statistically independent, i.e., asynchronous. The left quantity is the EB of the source obtained by coalescing the two sources by coupling state transitions. The next section will show that the combined EB of the two asynchronous sources is simply the sum of their individual EBs. Hence the result in (41) shows that coupling always increases the EB. This result is important for admission control, traffic shaping and policing and in the use of the EB concept for determining prices of network services.

Since in this subsection $\boldsymbol{M}$, B, and p are held fixed and only $\boldsymbol{\lambda}$ is varied, it is convenient to write for the EB.

$$e(\boldsymbol{\lambda}) = g_1(\boldsymbol{\lambda}) \tag{42}$$

It is understand that in this subsection $g_1(\lambda)$ denotes the maximal real eigenvalue of $[\mathbf{R} - \frac{1}{\zeta}\boldsymbol{M}]$, where $\zeta = \frac{\log p}{B}$. Note that $e(0) = 0$.

Proposition 7 $e(\lambda)$ is monotonic, increasing when any rate λ_i increases. Also $e(\lambda)$ is convex in $\lambda_1, \lambda_2, \ldots, \lambda_N$.

Proof : Strict Monotonicity follows from Result 4 of Section 2.3 since $[\boldsymbol{\Lambda} - \frac{1}{\zeta}\boldsymbol{M}]$ is an essentially nonnegative and irreducible matrix, and increasing any λ_i increases a diagonal element of the matrix . Strict convexity follows from Result 3 and a similar observation.

An immediate implication of the convexity of $e(\lambda)$ is that, for any two nonnegative (element wise) and non null rate vectors λ_1 and λ_2,

$$e(a\lambda_1 + (1-a)\lambda_2) \leq ae(\lambda_1) + (1-a)e(\lambda_2) \quad (0<a<1) \tag{43}$$

with equality holding if and only if all elements of $(\boldsymbol{\lambda}_1 - \boldsymbol{\lambda}_2)$ are identical.

Yet another implication is

Proposition III.8 For all $a \epsilon (0,1)$,

$$e(\lambda) > e(a\lambda) + e((1-a)\lambda) \tag{44}$$

proof:

$$e(\lambda) - e(a\lambda) = \int_0^1 g_1'\left(a\lambda + \tau(1-a)\lambda\right)d\tau$$

$$> \int_0^1 g_1'\left(\tau(1-a)\lambda\right)d\tau$$

$$= g_1\left((1-a)\lambda\right)$$

$$= e((1-a)\lambda).$$

Note that arbitrary splitting of the source with rate vector $\boldsymbol{\lambda}$ into two asynchronous sources with rates λ_1 and λ_2 ($\lambda_1 + \lambda_2 = \lambda$) do not generally preserve the inequality in (44). In fact, the following example shows that the reserve is not uncommon. Consider ON/OFF sources with exponentially distributed ON and OFF periods, for which the generator $\boldsymbol{M}$ is given in (32). Let the source rate vector $\boldsymbol{\lambda} = (r \;\; r)$ in which case the EB calculated from (33) is r. Now consider

$$\boldsymbol{\lambda}_1 = (0\; r) \text{ and } \boldsymbol{\lambda}_2 = (r\; 0) \quad (45)$$

The reader may verify that

$$e(\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2) < e(\boldsymbol{\lambda}_1) + e(\boldsymbol{\lambda}_2) \quad (46)$$

for all $\zeta < -\dfrac{|\beta-\alpha|}{r}$.

4. MULTIPLE SOURCES

We extend the results in the preceding section to multiplexing systems with several sources. First, we consider K arbitrary Semi Markov- modulated fluid sources and seek a characterization of the sources for which the admission criterion $\{G(B) \leq p\}$ is satisfied. As in the last section, the framework is asymptotic and the natural scaling in (37) is used. A key element of the asymptotic analysis here,as in Section 3.2, is the monotonicity of the maximal real eigenvalue with respect to the parameter in the inverse eigenvalue problem. The essential new element here is the simple additive form of the equation having K terms (called here the "coupled eigenvalue problem"), which is satisfied by the eigenvalues of system. This, together with the accompanying result which represents the system eigenvector as the Kronecker product of $\boldsymbol{K}$ low-order eigenvectors, constitutes a major decomposition of the eigenvalue problem. The algebraic theory which gives the decomposition is based on Kronecker representations and separability, and its antecedents are the results in Mitra *etal*.[32], Kosten [26],[27], Mitra [29] and Elwalid and Stern [14].

4.1 Representations

We suppose that there are K sources characterized by $(\boldsymbol{M}^{(k)}, \boldsymbol{\lambda}^{(k)})$ (k =1,2………K). Assume that for every k, source k has $N^{(k)}$ states and the generator $\boldsymbol{M}^{(k)}$ is irreducible. Let $\boldsymbol{R}^{(k)}$ = diag $(\boldsymbol{\lambda}^{(k)})$. $S^{(k)} = \{1,2,\ldots\ldots\ldots\ldots N^{(k)}\}$ is the state space of source k.

The aggregate source is a continuous-time Semi Markov chain with state space $S = \{ s / s = (s^{(1)},\ldots. S^{(k)}),\; s^{(k)} \in S^{(k)}, 1 \leq k \leq K \}$. The states of the sources are statistically independent and consequently the infinitesimal generator of the aggregate source is $\boldsymbol{M}$, where

$$\boldsymbol{M} = \boldsymbol{M}^{(1)} \otimes \boldsymbol{I} \otimes \ldots\ldots \otimes \boldsymbol{I} + \boldsymbol{I} \otimes \boldsymbol{M}^{(2)} \otimes \boldsymbol{I} \otimes \ldots \otimes \boldsymbol{I} + \ldots + \ldots\ldots \otimes \boldsymbol{I} \otimes \boldsymbol{M}^{(k)} \quad (47)$$

and $\otimes$denotes the Kronecker product. The Result 5 gives information on definitions and results of Kronecker algebra which are used in this paper. The standard compact representation of the form in (47) is

$$\boldsymbol{M} = \boldsymbol{M}^{(1)} \oplus \boldsymbol{M}^{(2)} \oplus \ldots\ldots \oplus \boldsymbol{M}^{(k)} \quad (48)$$

a K-fold Kronecker sum. The generator $\boldsymbol{M}$ is also irreducible.

The stationary probability vector of the aggregate source $\boldsymbol{\omega}$ is the Kronecker product of the stationary probability vectors of the individual sources. That is $\boldsymbol{\omega M} = 0$ and $\langle \boldsymbol{\omega}, \boldsymbol{1} \rangle = 1$ where

$$\boldsymbol{\omega} = \boldsymbol{\omega}^{(1)} \otimes \boldsymbol{\omega}^{(2)} \otimes \ldots\ldots \otimes \boldsymbol{\omega}^{(k)} \quad (49)$$

And $\boldsymbol{\omega}^{(k)} \boldsymbol{M}^{(k)} = 0$, $\langle \boldsymbol{\omega}^{(k)}, 1 \rangle = 1$. The system rate matrix $\mathbf{R}$ =is

$$\mathbf{R} \triangleq \mathbf{R}^{(1)} \oplus \boldsymbol{R}^{(2)} \oplus \ldots\ldots \oplus \mathbf{R}^{(k)} \quad (50)$$

The system drift matrix $\boldsymbol{D}$ is $\boldsymbol{D} \triangleq \mathbf{R} - c\boldsymbol{I}$ (51)

The ergodicity condition is $\bar{\lambda} < c$, where the mean rate of the aggregate source

$$\bar{\lambda} \triangleq \sum_k \overline{\lambda}^{(k)} = \sum_k \left(\lambda^{(k)}, \omega^{(k)}\right) \quad (52)$$

The peak rate of the aggregate source

$$\hat{\lambda} \triangleq \sum_k \hat{\lambda}^{(k)} \quad (53)$$

To avoid trivialities, we assume that $c < \bar{\lambda}$.

Result 5 The Kronecker product $\boldsymbol{A} \otimes \boldsymbol{B}$ of the matrix $\boldsymbol{A}$ of dimension p x q and the matrix B of dimension m x n is the matrix of dimension pm x qn obtained by replacing each element a_{ij} of the matrix $\boldsymbol{A}$ by the full matrix $\boldsymbol{a_{ij} B}$. (see, for example, [4].)

The Kronecker sum of $\boldsymbol{A}$ (n x n) and $\boldsymbol{B}$(m x m) denoted by $\boldsymbol{A} \oplus \boldsymbol{B}$ is defined as

$$\boldsymbol{A} \oplus \boldsymbol{B} = \boldsymbol{A} \otimes \boldsymbol{I}_m + \boldsymbol{I}_n \otimes \boldsymbol{B}$$

Where I_m and I_n are the identity matrices of order m and n, respectively. The operation $\otimes$ is associative but not commutative, and the same holds true for $\oplus$.

The following properties, which are proven in [4], [6], and [18], are used in this paper. All matrices (vectors) are assumed to be of appropriate order.

1) ***(A⊗B) (C⊗D) = (AC) ⊗ (BD).***
2) Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalue of the matrix $\boldsymbol{A}$ with corresponding eigenvectors $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \ldots, \boldsymbol{\alpha}_n$ and let $\mu_1, \mu_2, \ldots, \mu_m$ be the eigenvalues of $\boldsymbol{B}$ with corresponding eigenvectors $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_m$. Then the eigenvalues of A ⊕ B are the nm sums $\lambda_i + \mu_j$ which corresponding eigenvectors $\boldsymbol{\alpha}_i \otimes \boldsymbol{\beta}_j$, I = 1, 2, …, n and j = 1, 2, ………, m.

4.2 Decompositions

Let us transform the eigenvalue equation $z\,\boldsymbol{\phi} D = \boldsymbol{\phi} M$ to obtain the form of the inverse eigenvalue equation:

$$c\,\boldsymbol{\phi} = \boldsymbol{\phi} A(z) \tag{54}$$

Where

$$A(z) \triangleq \mathbf{R} - \frac{1}{z}\boldsymbol{M} \tag{55}$$

A key observation is that $A(z)$, like $\mathbf{R}$ and $\boldsymbol{M}$, also has the Kronecker sum form, which the reader is invited to verify:

$$A(z) = A^{(1)}(z) \oplus A^{(2)}(z) \oplus \ldots\ldots \oplus A^{(K)}(z) \tag{56}$$

Where

$$A^{(k)}(z) \triangleq \mathbf{R}^{(k)} - \frac{1}{z}\boldsymbol{M}^{(k)} \quad (1 \le k \le K) \tag{57}$$

From the eigenvalue and eigenvector results of Kronecker sums which are stated in the Result 5, we obtain

Proposition 9 A necessary and sufficient condition for ($c,\boldsymbol{\phi}$) to be a solution to the eigenvalue problem in (54) and thus also for (z,) to be a solution to the eigenvalue problem in (6) is

$$g^{(k)}(z)\,\boldsymbol{\phi}^{(k)}(z) = \boldsymbol{\phi}^{(k)}(z)\,A^{(k)}(z) \quad (1 \le k \le K) \tag{58a}$$

$$\textstyle\sum_k g^{(k)}(z) = c \tag{58b}$$

Where the eigenvector

$$\boldsymbol{\phi} = \boldsymbol{\phi}^{(1)} \otimes \boldsymbol{\phi}^{(2)} \otimes \ldots\ldots \otimes \boldsymbol{\phi}^{(K)} \tag{59}$$

We have called (58) the "coupled eigenvalue problem "since it is a system of K eigenvalue problems in which the dimensions are only $N^{(k)}$ ($1 \le k \le K$), and (58b) couples the constituent problems. As an alternative to this formal approach, the reader is invited to postulate the form for the eigenvector in (59) and to verify that the eigenvalue equation is satisfied if (58) holds. The remaining necessity part of the proof consists of verifying that the right number of eigenvalues are obtained by this procedure. For given z and k, there are $N^{(k)}$ solutions to (58a). Denote these by $g_{i(k)}^{(k)}(z)$, where $i(k) \in \{1,2,\ldots\ldots, N^{(k)}\}$. Hence, (58b) is equivalent to the family of equations

$$\sum_{k=1}^{K} g_{i(k)}^{(k)}(z) = c \tag{60}$$

in which all combinations of the subscripts are to be considered.

4.3 Maximal Real Eigen value

On examining the individual equations in(58a), we see that they are in the form of inverse eigenvalue problems, which are the subject of detailed investigation in Section III-A. It is known [see (19)] that, for $z < 0$, there exists a simple real solution $g_1^{(k)}(z)$, called the maximal real eigen value, such that

$$g_1^{(k)}(z) > \operatorname{Re}\, g_2^{(k)}(z) \ge \operatorname{Re}\, g_3^{(k)}(z) \ge \ldots \quad (z<0) \tag{61}$$

Moreover, it has been established in Propositions that $g_1^{(k)}(z)$ monotonically decreases from $\hat{\lambda}^{(k)}$ to $\bar{\lambda}^{(k)}$ as z increases form $-\infty$ to 0.

The analog of Proposition 4 is

Proposition 10 For $c \in (\bar{\lambda},\hat{\lambda})$, the dominant eigenvalue z_1 is the unique solution in $(-\infty, 0)$ to the equation.

$$\sum_{k=1}^{K} g_1^{(k)}(z_1) = c \tag{62}$$

i.e., the dominant eigenvalue is the unique parameter in $A^{(k)}(z)$ $(1 \le k \le K)$ such that the sum of their maximal real eigenvalue is c.

The proof closely parallels the proof of Proposition 4. The main items to note: (58) is satisfied by all eigenvalues; the dominance of $\sum g_1^{(k)}(z)$ for all $z < 0$ as reflected in (61),i.e.,

$$\sum_{k=1}^{K} g_1^{(k)}(z) \ge \sum_{k=1}^{K} \operatorname{Re} g_{i(k)}^{(k)}(z) \quad (z<0) \tag{63}$$

with equality holding only if $i(k) = 1$ for all k; the aforementioned monotonicity of $\sum g_1^{(k)}(z)$ and range $[\bar{\lambda}, \hat{\lambda}]$ for $z \in [-\infty, 0]$.

The analog of Proposition 5 also holds has a similar proof: the dominant eigenvalue z_1 is monotonic, strictly decreasing with increasing c for $c \in (\bar{\lambda}, \hat{\lambda})$.

4.4 Asymptotics

The asymptotic regime is specified by the scaling (37) in which the buffer size $B \to \infty$ and the buffer overflow probability $p \to 0$ in a manner parameterized by $\zeta \in [-\infty, 0]$. The following characterizes K sources which satisfy the admission criterion in this asymptotic regime.

Proposition 11 Suppose there are K sources $(\boldsymbol{M}^{(k)}, \boldsymbol{\Lambda}^{(k)})$($1 \le k \le K$). Let the admission criterion be $G(B) \le p$. Suppose $B \to \infty$ and $p \to 0$ in such a manner that

$\left(\frac{\log p}{B}\right) \to \zeta \in [-\infty, 0]$. If $\sum_k g_1^{(k)}(\zeta) < c$, then the admission criterion is satisfied. If $\sum_k g_1^{(k)}(\zeta) > c$, then the admission criterion is violated. Here $g_1^{(k)}(\zeta)$ is the maximal real Eigen Value of

$$A^{(k)}(\zeta) = [\mathbf{R}^{(k)} - \frac{1}{\zeta} M^{(k)}].$$

Proof: From (9), $G(B) = \sum_{i \geq 1} a_i \langle \phi_i, 1 \rangle e^{z_i B}$. Here, z_1 is the dominant eigenvalue, so that $z_1 > \operatorname{Re} z_i$ for all $i > 1$ and [see(62)] $\sum g_1^{(k)}(z_1) = c$. Hence,

$$\frac{G(B)}{p} = a_1 \langle \boldsymbol{\phi}_1, \mathbf{1} \rangle\, e^{(z_1 - \zeta)B} [1 + o(1)] \text{ as } B \to \infty \tag{64}$$

Now, from proposition 3, $\sum g_1^{(k)}(z)$ decreases as z increases; hence, if $\sum g_1^{(k)}(\zeta) < c$ then $z_1 < \zeta$ and $\left(\frac{G(B)}{p}\right) \to 0$ as $B \to \infty$ and the admission criterion is satisfied. Similarly, if $\sum g_1^{(k)}(\zeta) > c$ then $z_1 > \zeta$ and $\left(\frac{G(B)}{p}\right) \to \infty$ and the admission criterion is violated.

Now consider the implications of Proposition 11 on the admission control problem in which there are, say, J classes of sources. Every source of class $j (1 \leq j \leq J)$ is characterized by $(\boldsymbol{M}^{(j)}, \boldsymbol{\lambda}^{(j)})$. The problem is one of determining the set of all $\boldsymbol{K} = (K^{(1)}, K^{(2)}, \ldots\ldots\ldots, K^{(J)})$ for which the admission criterion $G_K(B) \leq p$ is satisfied, where $K^{(j)}$ is the number of sources of class j admitted to the multiplexing system.

Corollary., Let $A(B,p) = \{\boldsymbol{K} : G_K(B) \leq p\}$. Also let

$$\underline{A} \triangleq \left\{ K : \sum_j g_1^{(j)}(\zeta) K^{(j)} < c \right\}$$

$$\bar{A} \triangleq K : \sum_j g_1^{(j)}(\zeta) K^{(j)} < c$$

Where $g_1^{(j)}(\zeta)$ is the maximal real Eigen value of $[\boldsymbol{R}^{(j)} - \frac{1}{\zeta} \boldsymbol{M}^{(j)}]$. Then $\underline{A} \subseteq A(B,p) \subseteq \bar{A}$.

In applications of these asymptotic results, we approximate $A(B,p) \approx \{K : \sum g_1^{(j)}(\zeta) K^{(j)} < c\}$. Then, except for effects due to the integrality of $\boldsymbol{K}$, the acceptance set in $\boldsymbol{K}$ space is a simplex. The goodness of this approximation is the subject of numerical investigations in the next section.

Fig 2. The acceptance set for two classes of ON/OFF sources with $p = 2.06 \times 10^{-9}$

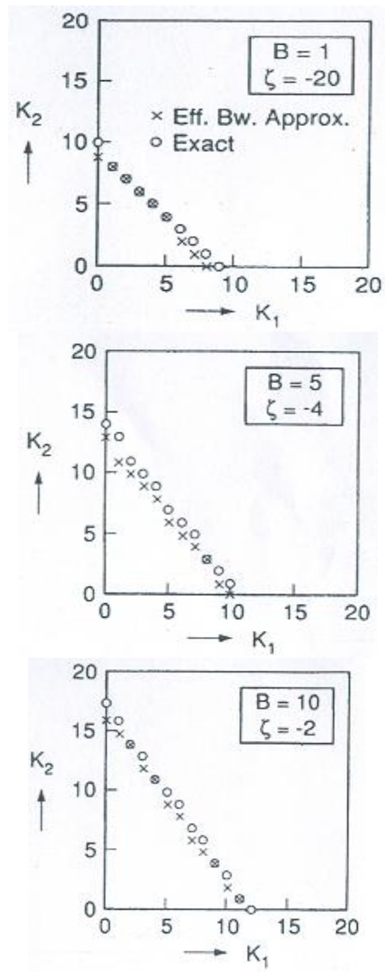
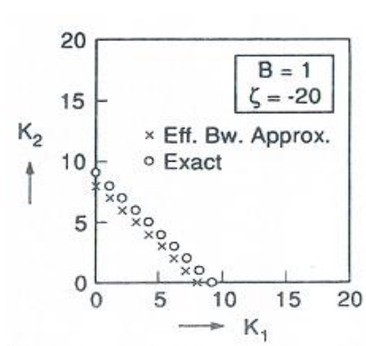


Fig 3. The acceptance set when the mean ON/OFF periods of both classes are half of those in Fig 1.



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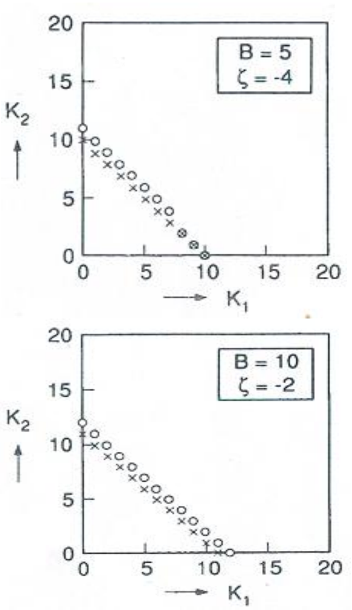
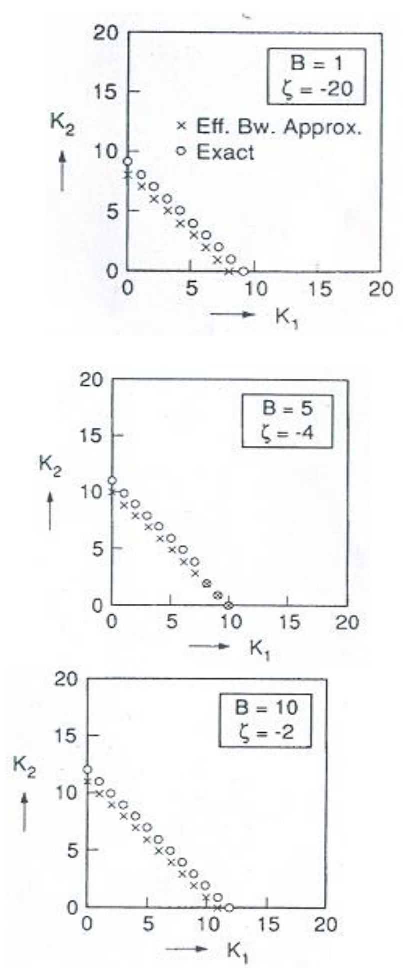


Fig 4. The acceptance set for two classes of sources with equal EBs and different mean and peak rates; p=2.06x10^{-9}



5. NUMERICAL STUDIES

In the previous sections, we showed that the EB of a source is a clearly defined and easily computed quantity. In this section, we numerically investigate three main issues:

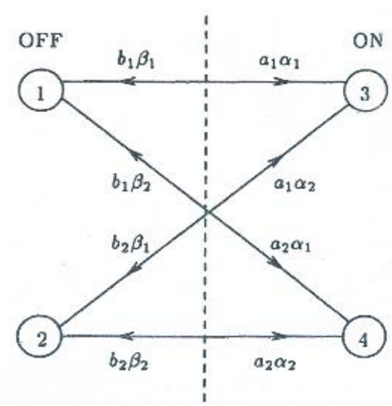
1. The accuracy of the EB when used in admission control and its sensitivity to source burstiness.
2. The effect of the variability of the ON and OFF periods on the EB.
3. The function of the Leaky Bucket regulator as a bandwidth- reducing device.

Figs. 2-4 address issue 1. Fig.2 displays the boundaries of the acceptance region for two source classes as computed from exact analysis and from using the EB approximation. Similar plots were obtained by Hunt and Gibbens [22]. The sources of both classes are ON/OFF with exponentially distributed ON and OFF periods. See (32) for an explanation of the source parameters, which are as follows:

Channel capacity $c = 8.43$

	α	β	λ_1	$\lambda_2 = \hat{\lambda}$
Class 1	1.0	1.0	0	1.0
Class 2	1.0	2.0	0	1.0

Fig 5. Four-state source model



$1/\alpha$ = mean off period

$1/\beta$ = mean on period

$C_v^2(\text{off})$ = squared coeff. of var. of off period

$C_v^2(\text{on})$ = squared coeff. of var. of on period

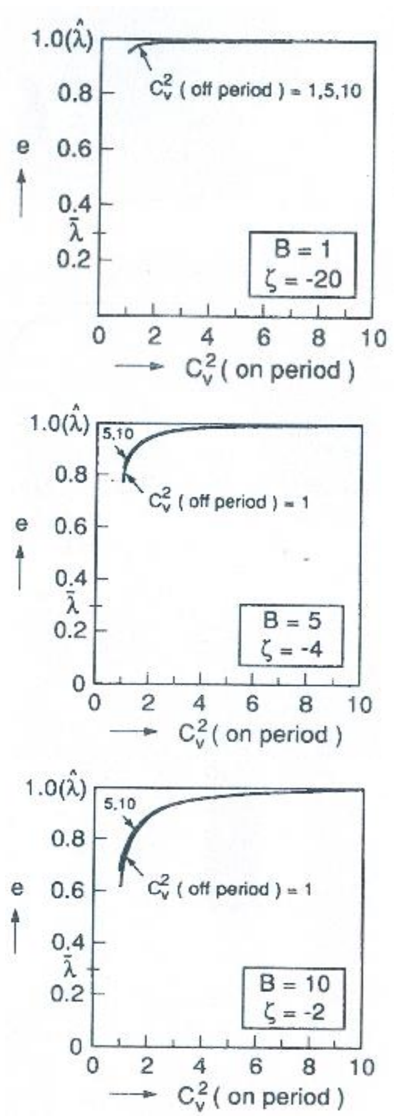
$$a_{1,2} = \frac{1}{2}\left[1 \pm \sqrt{\frac{C_v^2(\text{on})-1}{C_v^2(\text{on})+1}}\right]$$

$$b_{1,2} = \frac{1}{2}\left[1 \pm \sqrt{\frac{C_v^2(\text{off})-1}{C_v^2(\text{off})+1}}\right]$$

$\beta_1 = 2a_1\beta \quad \beta_2 = 2a_2\beta$

$\alpha_1 = 2b_1\alpha \quad \alpha_2 = 2b_2\alpha$

Fig 6. Effect of C_v^2 on ON and OFF periods, $\alpha = 0.4$, $\beta = 1$, $\hat{\lambda} = 1$; $p=2.06\times10^{-9}$



Hence the ON periods of the second class are only half as long. The buffer overflow probability $p = 2.06 \times 10^{-9}$. The buffer size B is varied: B = 1,5,10 which gives ζ = -20, -4, and -2, respectively. We point out that these plots differ from those in [20] in that p is constant; while in [22] p is varied with B to keep ζ constant. Also, in Figures 2-4 the data points are obtained by calculating the maximum acceptable value of K_2 for each value of K_1.

The figures observe that the EB approximation provides a conservative bound on the acceptance set.

In Fig. 3, the jitteriness of the sources is doubled, i.e., their mean ON and OFF periods are halved. Channel capacity, c= 8.43, as in Fig.2. The EB of each source decreases, resulting in an increase in the acceptance set. In Fig. 4, the two source classes have different mean and peak rates but the same EB. The parameters are the same as for Fig. 2, except that for class 2 the peak rate $\hat{\lambda}$= 1.05, 1.19, 1.29 for B=1, 5, 10, respectively. The symmetry in the plots confirms that EB provides an effective basis for admission control, while mean and peak source rates by themselves do not.

In the previous figures, the ON and OFF periods were assumed to be exponentially distributed. It is of interest to investigate the dependence of the EB on the variability of the ON and OFF periods. To generate distributions with a squared coefficient of variations larger than 1, we have chosen the hyperexponential distribution with balanced means. We consider a four-state source model [see Fig.5], where states 1 and 2 correspond to the OFF period and states 3 and 4 to the ON period. This model allows us (see the equations accompanying the figure) to vary the squared coefficient of variation of the OFF and ON periods, C_v^2(OFF) and C_v^2(ON), while keeping their means constant. In fig. 6, the EB of a source having parameters α= 0.4, β = 1, and $\hat{\lambda}$ = 1is plotted as a function of C_v^2(ON), for various values of C_v^2(OFF). We observe that the EB is sensitive to C_v^2(ON), and is less sensitive to C_v^2(OFF). Fig .7 displays similar behavior for a source with shorter ON and OFF periods.

In the context of rate-based congestion control, call admissions is necessarily complemented by traffic monitoring and regulation. The Leaky Bucket device can act as a traffic policer as well as a traffic shaper [5], [9], [12], [25], [30]. We consider the simplest form of the Leaky Bucket, which consists of a token pool of size B_T supplied with tokens at rate τ. In the model at hand, if an arriving cell finds the token buffer empty, it is marked, allowed into the network and treated thereafter as a low priority cell. We now examine the effect of the Leaky Bucket on the EB of a two-state ON/OFF source. To apply the results derived in this paper, we model the output stream of unmarked, i.e., high priority, cells leaving the Leaky Bucket as a three- state Semi Markov-modulated source as depicted in Fig..8 (see[12] for a detailed derivation). Fig. 9 plots EB versus B_T of different values for r and illustrates the bandwidth – reducing property of the Leaky Bucket. We let the unit of time be the mean length of the ON period and the unit of information be the amount generated by the source during an average ON period. Thus, the source peak rate and mean rate are equal to 1 and 0.286 units of information per unit of time, respectively. It is seen that the EB decreases from a maximum value equal to the source's original EB to a minimum value as B_T is decreased from five units of information to 0. The reduction in EB is alternatively, due to the increase in marking probability P_M which increases as B_T decreases, as shown in Fig. 9.

Fig 7. Effect of C_v^2 on ON and OFF periods, $\alpha = 2$, $\beta = 5$, $\hat{\lambda} = 1$; p=2.06x10^{-9}

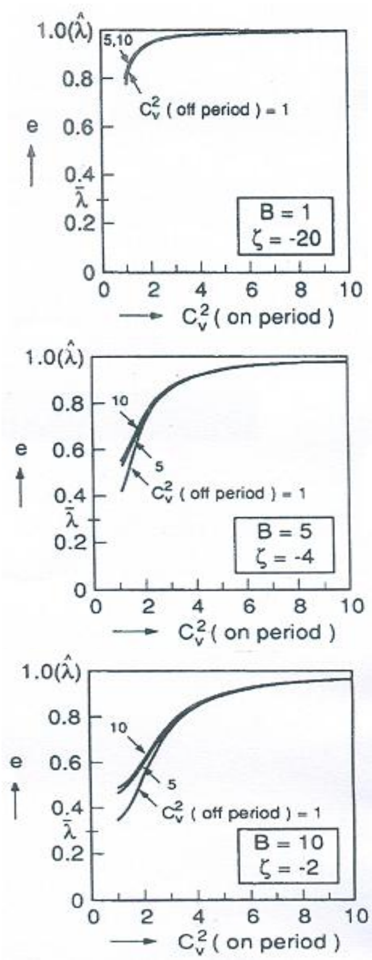
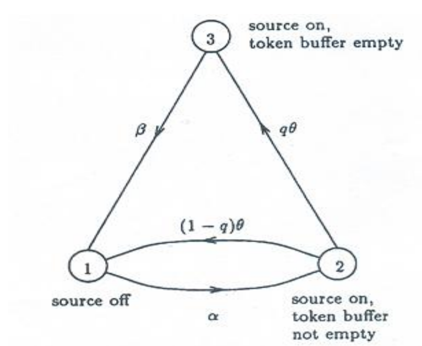
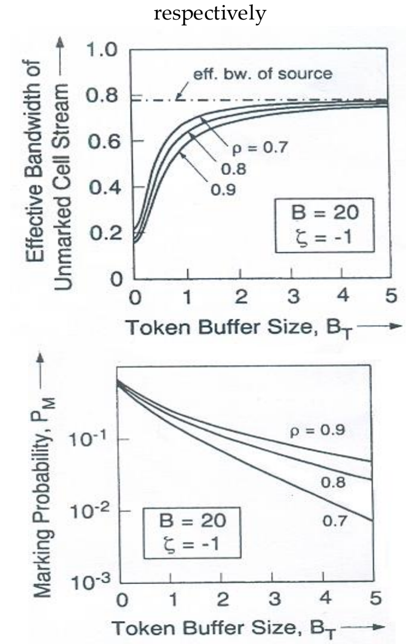


Fig 8. Approximation Markovian characterization of unmarked cell stream from Leaky Bucket regulator



$$\lambda = (0, \hat{\lambda}, r)$$
$$\theta = \beta\Delta/\{1 - \exp(zB_T)\}$$
$$q = 1 - \{1 - \exp(zB_T)\}/\Delta$$
$$\Delta = 1 - \frac{\alpha}{\beta}\,\frac{\hat{\lambda} - r}{r}\exp(zB_T)$$
$$z = -(\alpha + \beta)(1 - \rho)/(\hat{\lambda} - r)$$
$$\rho = \frac{\hat{\lambda}}{r}\,\frac{\alpha}{\alpha + \beta}$$
$$P_M = 1 - \frac{r}{\hat{\lambda}}\left[1 - \frac{(1 - \rho)}{\Delta}\right]$$

Fig 9. Effect of the Leaky Bucket regulator on EB $\alpha = 0.1$, $\beta = 0.25$, $\hat{\lambda} = 1$; p=2.06x10^{-9} .
Token rate r = 0.41, 0.35, 0.32 for regulator's ϱ = 0.7, 0.8, 0.9 respectively



6. EFFECTIVE BANDWIDTH OF A VIDEO TELECONFERENCING SERVICE TRAFFIC SOURCE.

In this section, we demonstrate that a realistic traffic source derived from measurements has an EB which is easy to calculate. The model, which is due to Tabatabai *et al.* [41], is for traffic from video teleconferencing services such as would be provided over VTST- based networks. Beginning with a 30-minute sequence of video teleconference data, the authors fit a variety of autoregressive and Semi Markov chain models and conclude that the only models sufficiently accurate for use in traffic studies is a multistate Markov chain model. A Version of this model which they recommend is the discrete autoregressive model DAR (1). While this model is structurally simple, the number of states is about 600 in the version which is used in their simulation experiments. We consider the continuous – time version of the DAR (1) model and show that its EB is readily calculated, even when the number of states is large.

In our model, the infinitesimal generator

$$M = \alpha[-\boldsymbol{I} + 1\langle\rangle f] \quad (65)$$

Where. ⟨⟩. denotes the outer product of vectors and in this case $\mathbf{1}\langle\rangle f$ is a matrix in which every row is identical to f. The form in (65) is obtained by identifying the transition matrix $\boldsymbol{P} = [\rho\boldsymbol{I} + (1-\rho)\,\mathbf{1}\langle\rangle f]$ in [41] with exp ($\boldsymbol{M\Delta}$), where ρ is a first- order autocorrelation coefficient, f is a probability vector, i.e., $f \geq 0$, $\langle f, 1\rangle = 1$, which is derived from the negative binominal distribution, and Δ is a small parameter associated with the time discretization. Since $\boldsymbol{P}$ = exp $(\boldsymbol{M\Delta}) \approx \boldsymbol{I} + \boldsymbol{M\Delta}$, (65) is obtained on identifying α with $(1-\rho)/\Delta$. The model in [23] implies a rate vector $\boldsymbol{\lambda}$ for our source with a linear dependence of λ_i on i. However, it will be equally convenient to let $\boldsymbol{\lambda}$ be arbitrary.

The EB of the source, $e(\boldsymbol{M}, \boldsymbol{\lambda}; B, p)$, is the maximal real solution e of the equation

$$\left|\mathbf{R} - \frac{1}{\zeta}\boldsymbol{M} - e\boldsymbol{I}\right| = 0 \quad (66)$$

where $\zeta = \dfrac{\log p}{B}$. To evaluate the determinant, we make use of the identity

$$|\boldsymbol{A} - \boldsymbol{a}\langle\rangle\boldsymbol{b}| = |\boldsymbol{A}|\,(1-\langle \boldsymbol{a}\boldsymbol{A}^{-1}, \boldsymbol{b}\rangle) \quad (67)$$

and obtain

$$\left|\boldsymbol{R} - \frac{1}{\zeta}\boldsymbol{M} - e\boldsymbol{I}\right| = \left[\prod_i\left(\lambda_i - e + \frac{\alpha}{\zeta}\right)\right]\left(1 - \frac{\alpha}{\zeta}\sum_i \frac{f_i}{\lambda_i - e + \frac{\alpha}{\zeta}}\right) \quad (68)$$

Hence, the EB is the maximal real solution to the equation

$$\frac{\alpha}{\zeta}\sum_i \frac{f_i}{\lambda_i - e + \frac{\alpha}{\zeta}} = 1 \quad (69)$$

Suppose that $\lambda_1 < \lambda_2 < \ldots < \lambda_{N-1} < \lambda_N$. The function on the left is monotonic, increasing in each of the intervals $(-\infty, \lambda_1 + \alpha/\zeta)$, $(\lambda_1 + \alpha/\zeta, \lambda_2 + \alpha/\zeta), \ldots$, $(\lambda_{N-1} + \alpha/\zeta, \lambda_N + \alpha/\zeta)$, $(\lambda_N + \alpha/\zeta, \infty)$; the function approaches $\pm\infty$ as the singularities at $(\lambda_i + \alpha/\zeta)$ are approached from the left and right, and approaches 0 as $e \to -\infty$ ans as $e \to \infty$. Hence,

Proposition 12 The EB of source $(\boldsymbol{M}, \boldsymbol{\lambda})$, where $\boldsymbol{M}$ is given in (65), is the unique real solution e in the interval $((\lambda_{N-1} + /\zeta, \lambda_N + \alpha/\zeta)$ to (69).

The use of this result in evaluating the effect of smoothing on the statistical multiplexing gain is the subject of current investigations.

7. EFFECTIVE BANDWIDTH OF SEMI MARKOV MODULATED POISSON SOURCES

In this section, we show that there are closely related concepts and arguments that apply to queues of jobs or packets. This parallelism between fluid flow and point processes has been noted before in [13] and [40], in particular for the decompositions of the eigenvalue problem. We shall show here that the parallelism also extends to the inverse eigenvalue problem, the qualitative properties of the maximal real eigenvalue as a function of the parameter of the problem and the concept of EB.

We begin, as in Section 3, with a single source $(\mathbf{M}, \boldsymbol{\lambda})$, where $\boldsymbol{M}$ is the irreducible infinitesimal generator of a controlling Semi Markov chain. The source emits packets in a Poisson stream at rate λ_s when in state s $(s \in S)$. Let $\mathbf{R}$ = diag $(\boldsymbol{\lambda})$. The packet length is exponentially distributed. The server is the output channel to the multiplexing buffer and has constant capacity or rate. The rate parameter μ is the ratio of the channel capacity to the mean packet length. The vector $\boldsymbol{\omega}$, the mean source rate $\bar{\lambda}$ and the peak rate $\hat{\lambda}$ are all defined as in Section 2.1. The ergodicity condition is $\bar{\lambda} < \mu$.

Let the stationary state distribution of the multiplexing system be denoted by $\boldsymbol{p}(n) = \{p_s(n) | s \in S|\}$, where

$$p_s(n) = \mathrm{P}\,(\Sigma = s, \mathrm{X} = n) \quad (s \in S\,;\, n = 0, 1, \ldots) \quad (70)$$

The balance equations are

$$0 = \boldsymbol{p}(n)\,[\boldsymbol{M} - \mathbf{R}] + \mu\boldsymbol{p}(n+1) \quad (n = 0)$$
$$= \boldsymbol{p}(n-1)\,\mathbf{R} + \boldsymbol{p}(n)[\boldsymbol{M} - \mathbf{R} - \mu\boldsymbol{I}] + \mu\boldsymbol{p}(n+1) \quad (n \geq 1) \quad (71)$$

The spectral representation of the solution to the balance equations

$$\boldsymbol{p}(n) = \sum_{i:|z|<1} a_i \phi_i z_i^n \quad (n \geq 0) \quad (72)$$

where $(z_i, \boldsymbol{\phi}_i)$ is an eigenvalue/eigenvector pair satisfying the eigenvalue equation

$$\boldsymbol{\phi}[\mu z^2\boldsymbol{I} + z(\boldsymbol{M} - \mathbf{R} - \mu\boldsymbol{I}) + \mathbf{R}]=0 \qquad (73)$$

Let the eigenvalues with modulus less than unity be indexed so

$$1 > |z_1| \geq |z_2| \geq \dots \qquad (74)$$

The coefficients $\{a_i\}$ in (72) are obtained from the normalization conditions Σ $p(n) = \boldsymbol{\omega}$. On substituting the solution in (72), we obtain

$$p(n) = \boldsymbol{\omega}\ [\boldsymbol{I}\text{-}\ \boldsymbol{R}]\ \boldsymbol{R}^n \qquad (n \geq 0) \qquad (75)$$

where $\boldsymbol{R} \triangleq \boldsymbol{\phi}^{-1}\mathbf{Z}\boldsymbol{\phi}$, (76)

$\boldsymbol{Z}$ = diag $(z_1, z_2, \dots)$ and $\boldsymbol{\phi}$ is the matrix with rows $\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \dots$. From (75),

$$G(n) \triangleq \mathrm{P}\ (X \geq n) = \langle \boldsymbol{\omega}\boldsymbol{R}^n, \mathbf{1}\rangle \qquad (n \geq 0) \qquad (77)$$

Equation (75) is also the well-known matrix-geometric from due to Neuts [33], who has shown that the Rate matrix $\boldsymbol{R}$ has spectral radius less than unity and is the minimal nonnegative solution to a matrix quadratic equation. Hence, $\boldsymbol{R}$ has a Perron root, the eigenvalue of maximum modulus which is real, simple, and in (0,1). From the spectral expansion of $\boldsymbol{R}$ in (76), we infer that the Perron root is z_1,the system eigenvalue[see (73) and (74)]. Hence z_1 is real, simple, and in (0,1). Since $z_1 > |z_i|$ for all $i > 1$, z_1 is called the dominant eigenvalue.

The inverse eigenvalue problem is obtained as in Section II- B. On writing $g(z) = \mu$

$$g(z)\ \boldsymbol{\phi} = \boldsymbol{\phi}\ A(z) \qquad (78)$$

Where $A(z) \triangleq \frac{1}{z}\ \mathbf{R} + \frac{1}{1-z}\ \boldsymbol{M}$ (79)

$A(z)$ is irreducible and essentially nonnegative for $z \in (0,1)$. Let $g_1(z)$ be its maximal real eigenvalue. We have proven

Proposition 13

1) $g_1(z) \sim \hat{\lambda}/z$ as z approaches 0 from the right, and $g_1(1) = \bar{\lambda}$.
2) $g'_1(z) < 0$ for $z \in (0,1)$.
3) The dominant eigenvalue z_1 is the unique solution in (0,1) satisfying $g_1(z_1) = \mu$

We next examine the admission criterion $\{G(B) \leq p\}$ in the asymptotic regime of large buffers B and small overflow probabilities p. Our result is

Proposition 14 Suppose $B \to \infty$ and $p \to 0$ in such a manner that $\dfrac{\log p}{B} \to \zeta \ \in [-\infty, 0]$. If $g_1\left(e^{\zeta}\right) < \mu$, then the admission criterion is satisfied. If $g_1\left(e^{\zeta}\right) > \mu$, then the admission criterion is violated.

Where $g_1\left(e^{\zeta}\right)$is the maximal real eigenvalue of

$$A(e^{\zeta}) = \frac{1}{e^{\zeta}}R + \frac{1}{1-e^{\zeta}}M\ .$$

On the basis of this result, the EB of the single source

$$e(\boldsymbol{M}, \boldsymbol{\lambda}; B, p) = g_1\left(e^{\zeta}\right) \qquad (80)$$

For a two-state SMMPP source with $(\boldsymbol{M}, \boldsymbol{\lambda})$ defined in (32),

$$g_1(z) = \frac{1}{2}\left(\frac{\lambda_1+\lambda_2}{z} - \frac{\alpha+\beta}{1-z}\right) - \frac{1}{2}\sqrt{\left(\frac{\lambda_1+\lambda_2}{z} - \frac{\alpha+\beta}{1-z}\right)^2 - 4\left(\frac{\lambda_1\lambda_2}{z^2} - \frac{\beta\lambda_1+\alpha\lambda_2}{z(1-z)}\right)} \qquad (81)$$

We next investigate, as in section 4, the decomposition of the expression in (80) When the source $(\mathbf{M}, \boldsymbol{\lambda})$ is the aggregate of K sources $(\boldsymbol{M}^{(k)}, \mathbf{A}^{(k)})$ $(1 \leq k \leq K)$. The key coupled eigenvalue problem in (58) carries over, with $A^{(k)}(z) \triangleq \frac{1}{z}\mathbf{R}^{(k)} + \frac{1}{1-z}\boldsymbol{M}^{(k)}$. With the benefit of proposition 13, we have proven

Proposition 15 Suppose there are K sources $(M^{(k)}, \lambda^{(k)})$ $(1 \leq k \leq K)$ supplying the multiplexing buffer. Let the admission criterion and asymptotic regime be as in proposition 14. If $\sum_k g_1^{(k)}\left(e^{\zeta}\right) < \mu$, then the admission criterion is satisfied. If $\sum_k g_1^{(k)}\left(e^{\zeta}\right) > \mu$, then the admission criterion is violated. Where $g_1^{(k)}\left(e^{\zeta}\right)$is the maximal real eigenvalue of $A^{(k)}\left(e^{\zeta}\right)$.

Table 1

The number of admissible sources obtained by the EB approximation and exact calculations

	Case 1		case 2	
μ	K_e	K^*	K_e	K^*
50	12	12	10	11
100	24	24	21	22
150	36	36	32	33
200	48	49	43	44
250	60	61	54	56
300	73	74	65	67

Thus, the simple additive structure to the EB of $\boldsymbol{K}$ sources exists in both the fluid and queuing frameworks. All the simplifying consequences discussed in Section 4.4 carry over.

We give numerical results on the use of the EB approximation to the admission control of a single class of two- state S**MMPP** sources. We consider two cases; Case1 and case2. The source in the two cases have the same mean rate ($\bar{\lambda}$ = 3.33) and different burstiness characteristics. In both cases, B = 200 and $p = 10^{-7}$, which gives e^{ζ} = 0.9226. The source parameters and EB $e = e\,(\boldsymbol{M}, \boldsymbol{\lambda}\,;\, \boldsymbol{B}, \boldsymbol{p})$ of the sources as computed by (80) and (81) are given below.

In Table I, we compare the admissible number of sources K_e, computed by EB approximation with K^*, obtained by exact calculations [36]. The comparison is carried out for a range of values of $\boldsymbol{\mu}$ the service rate.

	α	β	λ_1	$\lambda_2 = \hat{\lambda}$	e
Case 1	1	1	0	6.667	4.110
Case 2	1	4	0	16.667	4.568

The main observation in Table I is that $\boldsymbol{K}^*$ is tightly and conservatively bounded by K_e for sufficiently large B. Also, the admissible number in Case 1 is consistently larger than that of Case 2.This is because sources of Case 2 are more bursty.

In recent work [31], we have extended the results of this section to phase renewal (PH renewal) processes [33].

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